Name: ____ Period: ___

AP CALCULUS AB: Summer Work 2022

For students to successfully complete the objectives of the AP Calculus curriculum, the student must demonstrate a high level of independence, capability, dedication, and effort. This summer packet is not intended to scare you, but is intended to help you maintain/improve your skills. This packet is a ***requirement*** for those entering the AP Calculus AB course and is **due on the first day of class**. At that time, it will be collected by your teacher and graded. And we will be having a test/quiz on this material, likely on the second day of class. *If this packet is not completed by the first day of class, you should consider yourself behind;* not only on your class grade, but also on the concepts necessary for success in Calculus. Complete as much of this packet on your own as you can, then get together with a friend or use Google or YouTube to find help with the topic.

There is a formula sheet at the end of this review. Feel free to use this sheet on this packet. Please know that you will **not** be supplied with a formula sheet for tests or quizzes during the Calculus course.

Requirements: *The following are guidelines for completing the summer work packet...*

- There are 80 questions, some with multiple parts, that you must complete. You must **show all of your work clearly** (it should be neat and legible) on the packet or, if not enough room, on a separate sheet of paper.
- Be sure all problems are neatly organized and all writing is legible.
- In the event that you are unsure how to perform functions on your calculator, you may need to read through your calculator manual or use Google or YouTube to understand the necessary syntax or keystrokes. You must be familiar with certain built-in calculator functions such as finding values, intersection points, using tables, and finding zeros of a function.
- I expect you to come in with certain understandings that are prerequisite to Calculus. A list of these topical understandings is below.

Topical understandings within summer work...

- Manipulate algebraic expressions involving exponents and radicals.
- Manipulate algebraic fractions
- Factor algebraic expressions
- Solve equations through quadratics; complete the square in an algebraic expression
- Solve simultaneous equations
- Solve “word problems” (i.e. translate words into algebraic expressions)
- Functions and graphs (rectangular coordinates)
- Solve inequalities
- Find roots of polynomials using synthetic division
- Use binomial theorem
- Manipulate complex numbers
- Manipulate logarithmic expressions, graph logarithmic functions and solve logarithmic equations.
- Solve exponential equations
- Find equations of straight lines and conic sections
- Determine inverse functions

Trigonometry also plays an important role in calculus and is used throughout the course. The student must know trigonometry in order to be successful in the course. In particular, the student should be familiar with the following.

- Fundamental definitions
- Basic identities
- Application of basic identities to the solutions of trigonometric equations and proving identities
- Graphing trigonometric functions
- Radian measure
- The inverse trigonometric functions
- Domain and range of trigonometric and inverse trigonometric functions
- Complete understanding (and memorization) of the Unit Circle is a **must**!

Finally, I suggest not waiting until the last two weeks of summer to begin on this packet. If you spread it out, you will most likely retain the information much better. However, I strongly suggest reviewing the entire packet in the week before school starts. Once again this is due on the first day of class. Best of luck and if you have any questions, feel free to contact me at jdhillon@upatoday.com.

<u>Complex Fractions</u>

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \cdot \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{-\frac{2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{-\frac{2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \cdot \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x}$$

Simplify each of the following.

1. $\frac{x+1}{2-\frac{1}{x-1}}$

2. $\frac{9-x^{-2}}{3+x^{-1}}$

3. $\frac{2x}{x^2-6x+9} - \frac{1}{x+1} - \frac{8}{x^2-2x-3}$

4. If $f(x) = 3x^2+5x+1$, what is $\frac{f(x+h)-f(x)}{h}$?

Exponential, Logarithmic and Rational Expressions Simplify each of the following, without a calculator.

5. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

6. $\dfrac{4}{\sqrt{3}+\sqrt{5}}$ (Rationalize the denominator)

7. $2\log(x-3)+\log(x+2)-6\log x$

8. $\dfrac{x-4}{x^2-3x-4}$

9. ln 1

10. $\dfrac{5-x}{x^2-25}$

11. $e^{(1+\ln x)}$

12. $\dfrac{\frac{2}{x^2}}{\frac{10}{x^5}}$

13. Rationalize the denominator $\dfrac{\sqrt[3]{72}}{\sqrt[3]{9x^5}}$

14. $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$

15. $\log_{1/2} 8$

16. $x^{3/2}\left(x+x^{5/2}-x^2\right)$

17. $e^{3\ln x}$

18. $\log_{16}(\log 10{,}000)^3$

Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $\left(f \circ g\right)(x) = f(g(x))\ OR\ f[g(x)]$ read "***f*** of ***g*** of ***x***" means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $\boldsymbol{f(x) = -5x + 7}$ and $\boldsymbol{g(x) = 3x^2 + 9}$. Find each.

19. $f(-12) =$__________

20. $g(-3) =$__________

21. $g(t+2) =$ __________

22. $f\left[g(-2)\right] =$__________

23. $g\left[f(m+2)\right] =$__________

24. $\dfrac{f(x+h) - f(x)}{h} =$______

Factor Completely:

25. $16x^2 - 8x + 1$

26. $2x^3 - 5x^2 - 18x + 45$

27. $36x^2 - 100y^2$

28. $10x^2 + 13x - 3$

Equation of a line

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: y = c (slope is 0)

29. Find the equation of a line perpendicular to $2x - 3y = 5$ passing through the point (-16, 71).

30. Find the equation of a line passing through the points (-12, 4) and (6, 5).

Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

31. $f(x) = \frac{1}{x^2}$

32. $f(x) = \frac{x^2}{x^2 - 4}$

33. $f(x) = \frac{2 + x}{x^2(1 - x)}$

Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

34. $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

35. $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

36. $f(x) = \frac{4x^5}{x^2 - 7}$

Solve the following equations or inequalities: Solve these without a calculator; make sure to show your steps for solving them.

37. $4t^3 - 12t^2 + 8t = 0$

38. $3\sqrt{x-2} - 8 = 8$

39. $\log x + \log(x-3) = 1$

40. $\dfrac{x-5}{3-x} > 0$

41. $4e^{2x} = 5$

42. $\frac{3}{4} - \frac{1}{6x} = \frac{1}{x}$

43. $27^{2x} = 9^{x-3}$

44. $(x+1)^2(x-2) + (x+1)(x-2)^2 = 0$

45. $\ln 3x + \ln 3 = 3$

46. $\log_5(x+3) - \log_5 x = 2$

Trigonometric Equations:

Isolate the variable, sketch a reference triangle, find all the solutions within the domain $0 \le x < 2\pi$. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

47. $\sin x = -\dfrac{1}{2}$

48. $\cos x = \sin x$

49. $\cos^3 x = \cos x$

50. $\sin x - 2\sin x\cos x = 0$

51. $3\cot^2 x - 1 = 0$

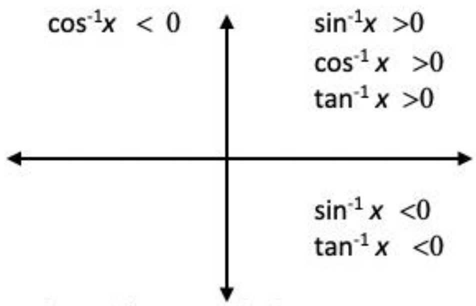
52. $4\cos^2 x - 3 = 0$

Inverse Trigonometric Functions:

Recall: Inverse Trig Functions can be written in one of two ways:

$$\arcsin(x) \qquad\qquad \sin^{-1}(x)$$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



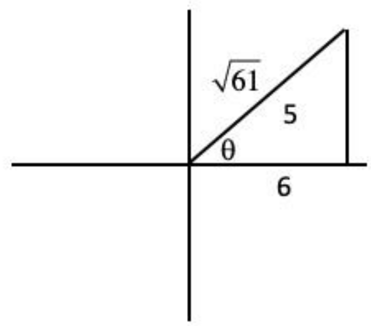
Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.



$$\cos\theta = \frac{6}{\sqrt{61}}$$

For each of the following, find the value without a calculator.

53. $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$

54. $y = \arccos(-1)$

55. $\cos\left(\sin^{-1}\frac{1}{2}\right)$

56. $\tan\left(\cos^{-1}\left(-\frac{2}{3}\right)\right)$

57. $\csc\left(\cos^{-1}\frac{12}{13}\right)$

58. $\sin\left(\arctan\frac{12}{5}\right)$

59. $\sin\left(\sin^{-1}\frac{7}{8}\right)$

Without a calculator, determine the exact value of each expression, if it exists.
YOU SHOULD HAVE THE UNIT CIRCLE COMPLETELY MEMORIZED AND MASTERED!!

60. $\sin 0$

61. $\cos\frac{7\pi}{6}$

62. $\cot\frac{\pi}{3}$

63. $\sin\frac{\pi}{2}$

64. $\sec\frac{5\pi}{3}$

65. $\tan\frac{\pi}{2}$

66. $\sin\frac{3\pi}{4}$

67. $\tan\frac{7\pi}{4}$

68. $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$

Domain and Range: Determine the domain of each function. Write your answer in interval notation.

69. $y = \sqrt{x^2 + 4}$ *Domain* ____________________

70. $g(x) = \frac{x - 2}{x^2 - 4}$ *Domain* ____________________

71. $y = \sqrt{16 - x^2}$ *Domain* ____________________

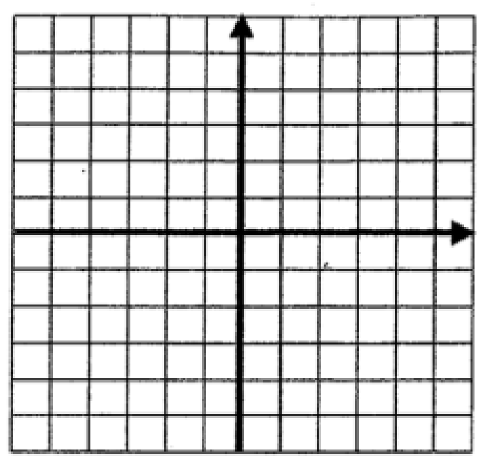
72. $y = \ln(2 - x)$ *Domain* ____________________

73. $h(x) = 2^x + 1$ *Domain* ____________________

74. $f(x) = \frac{\sqrt{3-x}}{x+1}$ *Domain* ____________________

Sketch the graph without a calculator.

75. $f(x) = \begin{cases} -x^2 & -2 \le x < 1 \\ -4 & x = 1 \\ 2x - 1 & 1 < x \le 5 \end{cases}$



Miscellaneous Topics

76. The number of elk after t years in a state park is modeled by the function $P(t)=\dfrac{1216}{1+75e^{-0.03t}}$.

    a. What was the initial population of elk?

    b. When will the number of elk be 750? *Round your answer to the nearest whole number and state the units.*

77. Use polynomial division to state the quotient and remainder of $\dfrac{2x^3-5x^2+5x-9}{x-2}$.

78. Use a graphing calculator to solve the equation for *x*. *Round your answer to 3 decimal places.*

    $e^{2x}=3x^2$.

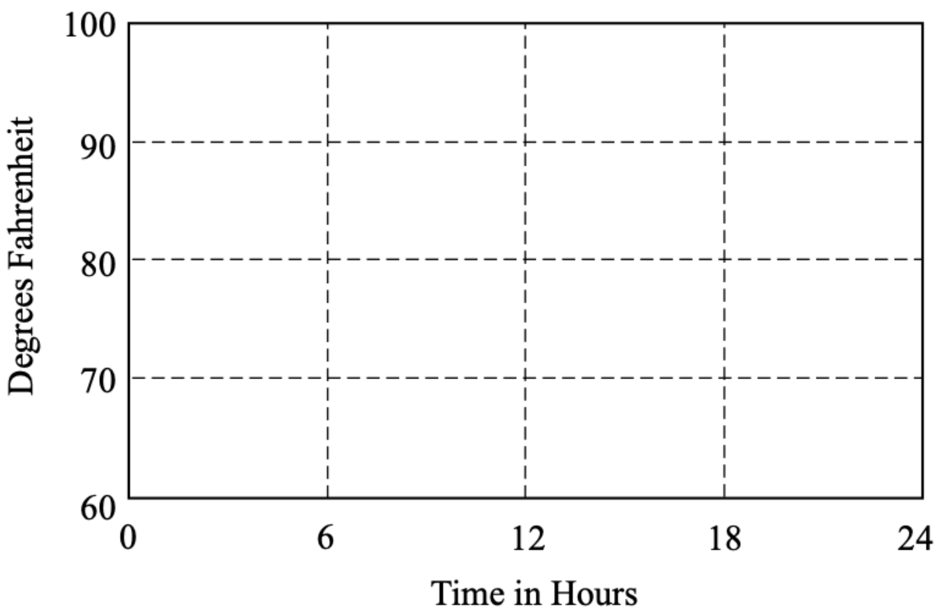
Free Response Question: Answer the following questions completely.

79. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right), \qquad \text{for } 0 \le t \le 24$$

where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours.

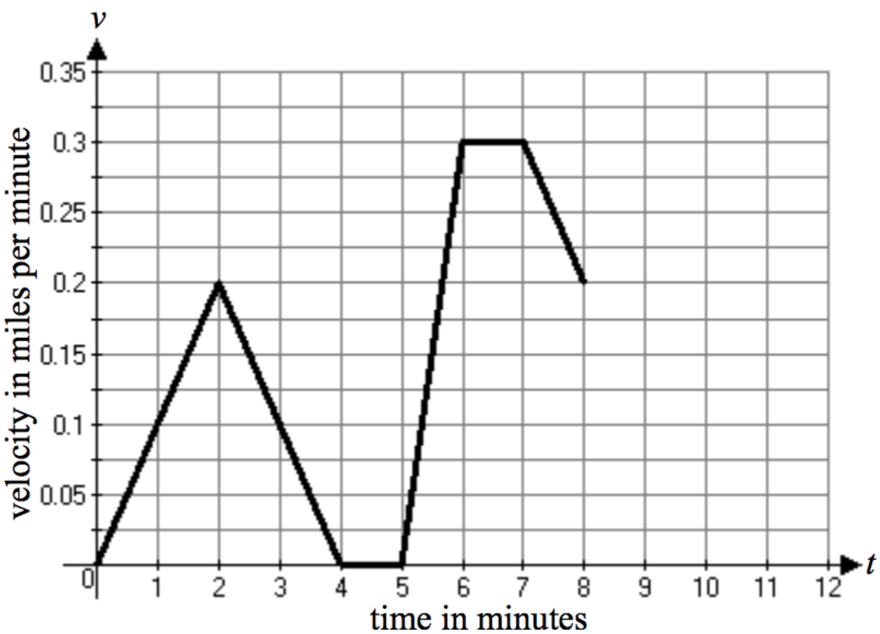
a. Sketch the graph of F on the grid below.



b. Define the amplitude, period, phase shift, and vertical shift of the function F.

c. Within the context of the problem, explain the meaning of the point $F(6) = 80$.

d. Find the average rate of change of temperature between $t = 6$ and $t = 14$.



t	$v(t)$
0	0
2	0.2
4	0
5	0
6	0.3
7	0.3
8	0.2

80. Eric rides his bike to school along a straight road starting from home at $t = 0$ minutes. For the first 8 minutes, Eric's velocity, in miles per minute, is modeled by the piecewise linear function whose graph is shown above.

a. How fast is Eric riding at $t = 2$ minutes? Include units in your answer.

b. Write an equation for the portion of the velocity function between $t = 2$ and $t = 4$ minutes. Use your equation to find Eric's velocity at $t = 3.8$ minutes.

c. Eric decreases his velocity from $t = 8$ to $t = 12$ at a constant rate so that his velocity at $t = 12$ is 0 miles/minute. On the graph, extend the velocity function to show this new information.

d. The area between the velocity graph and the x-axis represents the distance Eric travelled. Use your understanding of area of polygons to determine the total distance Eric travelled in 12 minutes.

Formula Sheet

All of these should be understood and memorized (unless otherwise indicated)

<u>Reciprocal Identities</u>: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

<u>Quotient Identities</u>: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

<u>Pythagorean Identities</u>: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

<u>Double Angle Identities</u>: (these do not need to be memorized)

$$\sin 2x = 2\sin x \cos x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

<u>Logarithms</u>: $y = \log_a x$ is equivalent to $x = a^y$

<u>Product property</u>: $\log_b mn = \log_b m + \log_b n$

<u>Quotient property</u>: $\log_b \frac{m}{n} = \log_b m - \log_b n$

<u>Power property</u>: $\log_b m^p = p\log_b m$

<u>Property of equality</u>: If $\log_b m = \log_b n$, then m = n

<u>Change of base formula</u>: (no need to memorize) $\log_a n = \frac{\log_b n}{\log_b a}$

<u>Derivative of a Function</u>: (no need to memorize, you will learn more about it this year)

Slope of a tangent line to a curve (or the derivative): $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

<u>Slope-intercept form</u>: $y = mx + b$

<u>Point-slope form</u>: $y - y_1 = m(x - x_1)$

<u>Standard form</u>: Ax + By + C = 0